Liquid Crystal Elastomer Dielectric Constant Measurements

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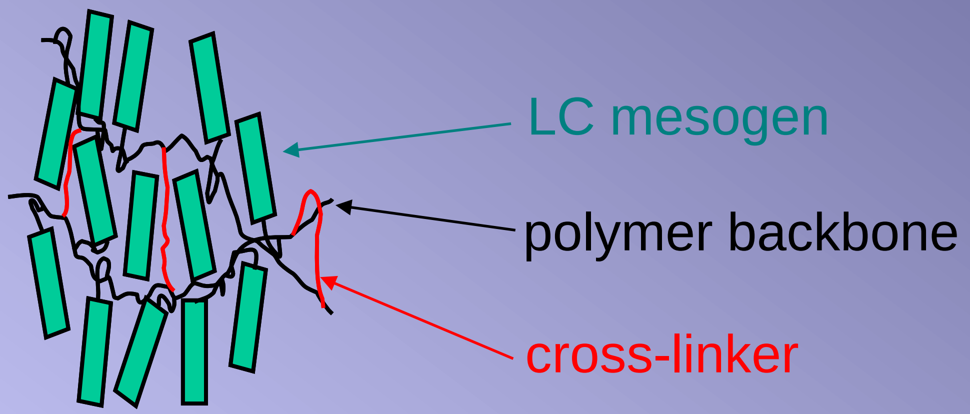
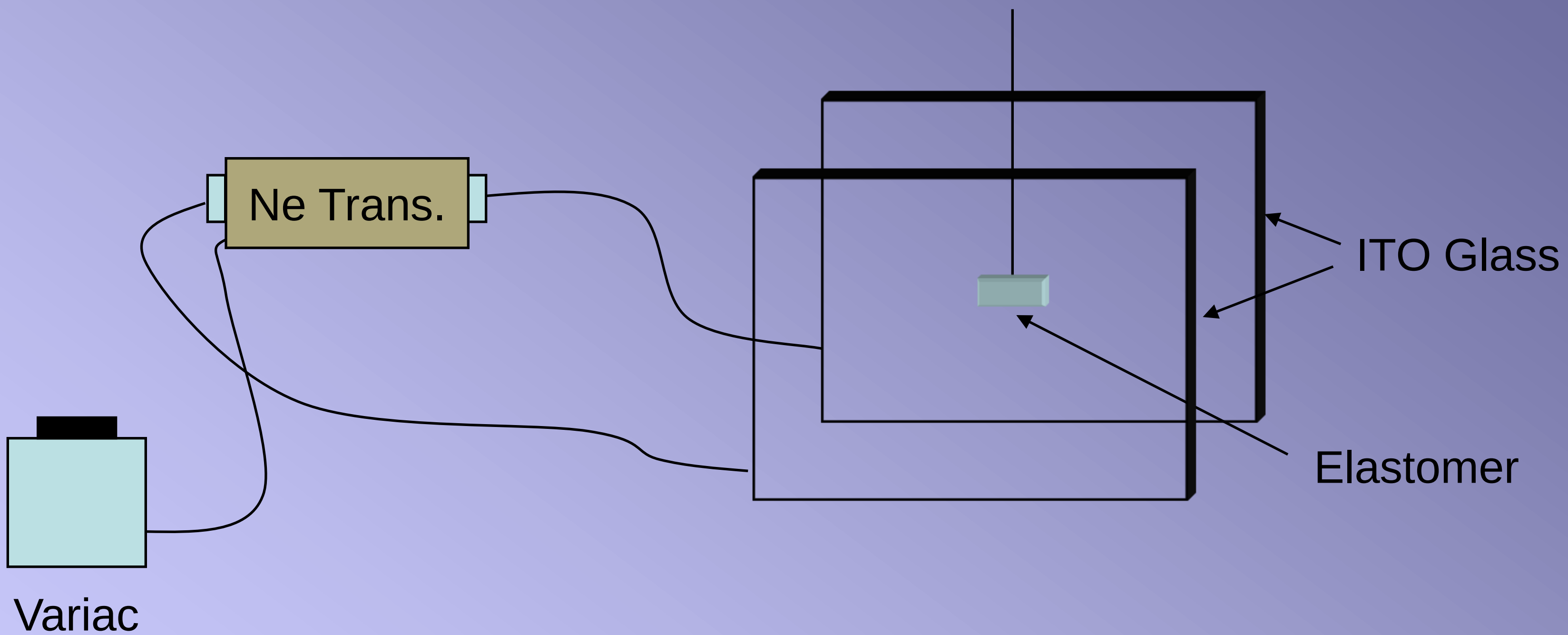
Liquid Crystal Elastomers

- liquid crystal + rubber



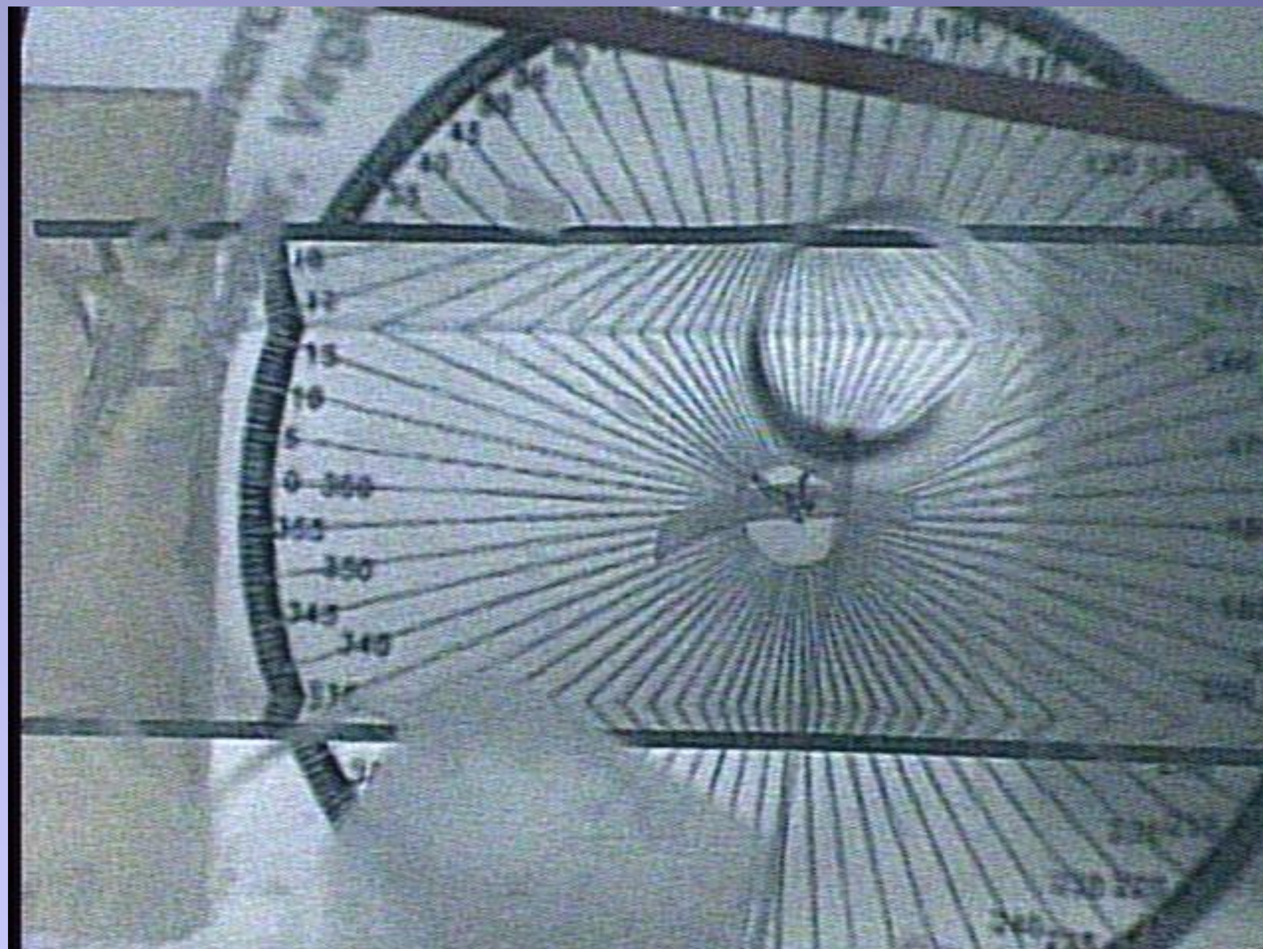
- LC nematic monodmain
- synthesize here - need to characterize

Experiment 1

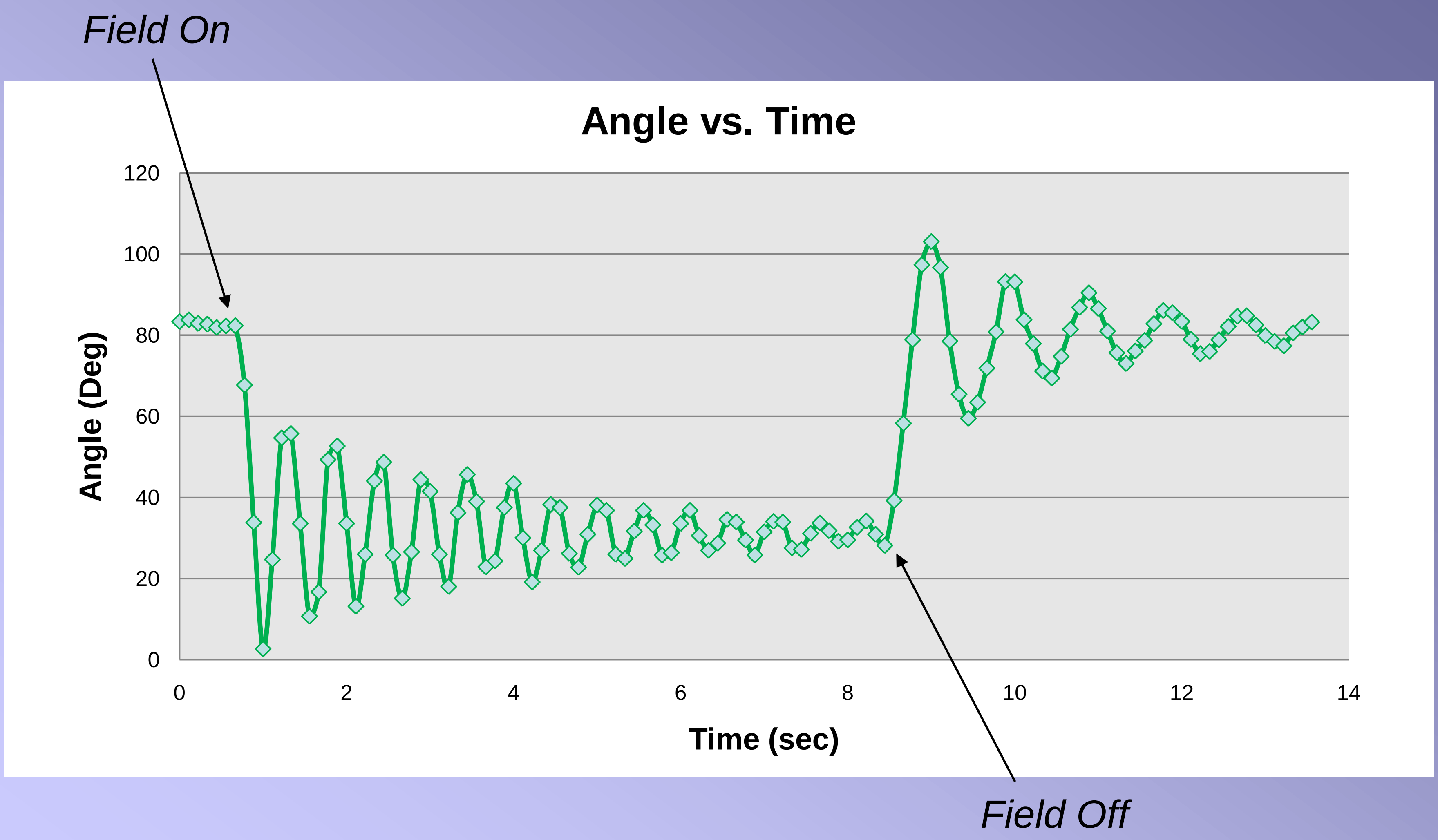


Elastomer is suspended on a string between two parallel pieces of ITO coated glass. Elastomer aligns along sufficiently strong applied field.

Video of Experiment



Experimental Data



Overview

Want to solve

$$\tau = I\frac{d^2\theta}{dt^2} = \frac{d\xi}{d\theta}$$

$\tau - torque$

$\xi - energy$

$I - moment\ of\ inertia$

Directly from experimental data

Depends on:

$$\left.\begin{matrix}\varepsilon_{\parallel} \\ \varepsilon_{\perp}\end{matrix}\right\} - \text{dielectric constant components}$$

k – *spring constant*

material properties

Theory

Energy of elastomer:

$$\xi = -\frac{V}{2}\vec{D}\cdot\vec{E} - \frac{1}{2}k\theta^2$$

$$= -\frac{V}{2}\left[\varepsilon_0\left(1+\chi\right)\vec{E}\right]\cdot\vec{E} - \frac{1}{2}k\theta^2$$

$$= -\frac{V}{2}\left[\varepsilon_0\vec{E} + \vec{P}\right]\cdot\vec{E} - \frac{1}{2}k\theta^2$$

orientationally invariant

$$= -\frac{V}{2}\left[\varepsilon_0 E^2 + \vec{P}\cdot\vec{E}\right] - \frac{1}{2}k\theta^2$$

$$= -\frac{V}{2}\vec{P}\cdot\vec{E} - \frac{1}{2}k\theta^2$$

k - *spring constant*

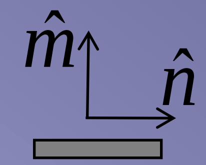
V - *volume*

χ – *susceptibility*

$\vec{P}$ – *polarization*

Theory

In general:



$$\vec{P} = \rho \overleftrightarrow{\alpha} \vec{E}^{loc}$$

$$= \rho\alpha_{ll} E_{ll}^{loc}\hat{n} + \rho\alpha_{\perp} E_{\perp}^{loc}\hat{m}$$

$$= \left(\rho\alpha_{ll}\hat{n}\hat{n} + \rho\alpha_{\perp}(\overleftrightarrow{I} - \hat{n}\hat{n})\right)\vec{E}^{loc}$$

$\alpha - polarizability$

$\rho - density$

where

$$\vec{E}_{loc} = \vec{E}_{app} - \mathrm{N}\frac{\vec{P}}{\varepsilon_0}$$

$N - depolarizing\ tensor$

$$= \left(\vec{E}_{app} - N_{ll}\frac{\vec{P}}{\varepsilon_0}\right)\hat{n}\hat{n} + \left(\vec{E}_{app} - N_{\perp}\frac{\vec{P}}{\varepsilon_0}\right)\left(\overleftrightarrow{I} - \hat{n}\hat{n}\right)$$

Theory

cont'd

$$\vec{P} = \rho\alpha_{ll}\left(\vec{E}_{app} - \mathrm{N}_{ll}\frac{\vec{P}}{\varepsilon_0}\right)\hat{n}\hat{n} + \rho\alpha_{\perp}\left(\vec{E}_{app} - \mathrm{N}_{\perp}\frac{\vec{P}}{\varepsilon_0}\right)\left(\overleftrightarrow{I} - \hat{n}\hat{n}\right)$$

$$\vec{P}\left(\overleftrightarrow{I}\left(1 + \frac{\alpha_{\perp}\mathrm{N}_{\perp}\rho}{\varepsilon_0}\right) + \frac{\rho}{\varepsilon_0}\left(\alpha_{ll}\mathrm{N}_{ll} - \alpha_{\perp}\mathrm{N}_{\perp}\right)\hat{n}\hat{n}\right) = \left(\left(\alpha_{ll} - \alpha_{\perp}\right)\hat{n}\hat{n} + \alpha_{\perp}\overleftrightarrow{I}\right)\rho\vec{E}_{app}$$

Make substitutions:

$$a = \left(1 + \frac{\alpha_{\perp}\mathrm{N}_{\perp}\rho}{\varepsilon_0}\right) \qquad b = \frac{\rho}{\varepsilon_0}\left(\alpha_{ll}\mathrm{N}_{ll} - \alpha_{\perp}\mathrm{N}_{\perp}\right)$$

then

$$\vec{P}\left(a\overleftrightarrow{I} + b\hat{n}\hat{n}\right) = \left(\left(\alpha_{ll} - \alpha_{\perp}\right)\hat{n}\hat{n} + \alpha_{\perp}\overleftrightarrow{I}\right)\rho\vec{E}_{app}$$

Theory

$$\vec{P}\left(a\overleftrightarrow{I}+b\hat{n}\hat{n}\right)=\left(\left(\alpha_{ll}-\alpha_{\perp}\right)\hat{n}\hat{n}+\alpha_{\perp}\overleftrightarrow{I}\right)\rho\vec{E}_{app}$$

Choose A & B such that

$$\left(a\overleftrightarrow{I}+b\hat{n}\hat{n}\right)\left(A\overleftrightarrow{I}+B\hat{n}\hat{n}\right)=\overleftrightarrow{I}$$

then

$$A=\frac{1}{a} \qquad B=-\frac{b}{a(a+b)} \qquad A+B=\frac{1}{a+b}$$

$$\vec{P}=\left(A\overleftrightarrow{I}+B\hat{n}\hat{n}\right)\left(\left(\alpha_{ll}-\alpha_{\perp}\right)\hat{n}\hat{n}+\alpha_{\perp}\overleftrightarrow{I}\right)\rho\vec{E}_{app}$$

$$\vec{P}=\left(\rho\alpha_{\perp}A\overleftrightarrow{I}+\left(B\rho\alpha_{\perp}+(A+B)\rho(\alpha_{ll}-\alpha_{\perp})\right)\hat{n}\hat{n}\right)\vec{E}_{app}$$

Theory

Recall

$$\xi = -\frac{V}{2}\vec{P}\cdot\vec{E} - \frac{1}{2}k\theta^2$$

Thus

orientationally invariant

$$\xi = -\frac{V}{2}\left[\rho g\alpha_{\perp}AE_{app}^2 + \left(B\rho\alpha_{\perp} + (A+B)\rho(\alpha_{ll} - \alpha_{\perp})\right)\cos^2\theta E_{app}^2\right] - \frac{1}{2}k\theta^2$$

$$= -\frac{V}{2}\left(B\rho\alpha_{\perp} + (A+B)\rho(\alpha_{ll} - \alpha_{\perp})\right)\cos^2\theta E_{app}^2 - \frac{1}{2}k\theta^2$$

Want to solve

$$\tau = I\frac{d^2\theta}{dt^2} = \frac{d\xi}{d\theta}$$

$\tau - torque$

$$\frac{d\xi}{d\theta} = \left(B\rho\alpha_{\perp} + (A+B)\rho(\alpha_{ll} - \alpha_{\perp})\right)V\cos\theta\sin\theta E_{app}^2 - k\theta$$

Theory

Also

$$\varepsilon_{ll} = 1 + \frac{\rho\alpha_{ll}}{\varepsilon_o}, \quad \varepsilon_{\perp} = 1 + \frac{\rho\alpha_{\perp}}{\varepsilon_o} \longrightarrow \alpha_{ll} - \alpha_{\perp} = \frac{\varepsilon_0(\varepsilon_{ll} - \varepsilon_{\perp})}{\rho}$$

So

$$I\frac{d^2\theta}{dt^2} = \frac{d\xi}{d\theta} = \Big(B(\varepsilon_{\perp} - 1) + \big(A + B\big)\big(\varepsilon_{ll} - \varepsilon_{\perp}\big)\Big)V\varepsilon_0 \cos\theta \sin\theta E_{app}^2 - k\theta$$

where

$$B = \frac{(\varepsilon_{\perp} - 1)N_{\perp} - (\varepsilon_{ll} - 1)N_{ll}}{(1 + (\varepsilon_{ll} - 1)N_{ll})(1 + (\varepsilon_{\perp} - 1)N_{\perp})} \qquad A + B = \frac{1}{1 + (\varepsilon_{ll} - 1)N_{ll}}$$

Need the following:

$N_{\parallel}, N_{\perp} - depolarizing\ factors$

$E_{app} - electric\ field$

$I - moment\ of\ inertia$

$k, \varepsilon_{\perp}, or\ \varepsilon_{\parallel} - free\ parameters$

Depolarizing Factors

Assume depolarizing factors for an ellipse can be used:

$$N_\alpha = \frac{a+b+c}{2}\int_0^\infty \frac{ds}{(\alpha^2+s)\sqrt{(a^2+s)(b^2+s)(c^2+s)}}$$

$$\alpha = a, b, c \qquad ellipse\ semi-axes$$

$$a = 10mm \qquad\qquad N_{10} = N_{ll} = .0076$$

$$b = 1.9mm \longrightarrow N_{1.9} = .0922$$

$$c = .22mm \qquad\qquad N_{.2} = N_{\perp} = .9002$$

Elastomer Properties

$$l = 2.00\,cm$$

$$w = 3.8\,mm$$

$$d = .43\,mm$$

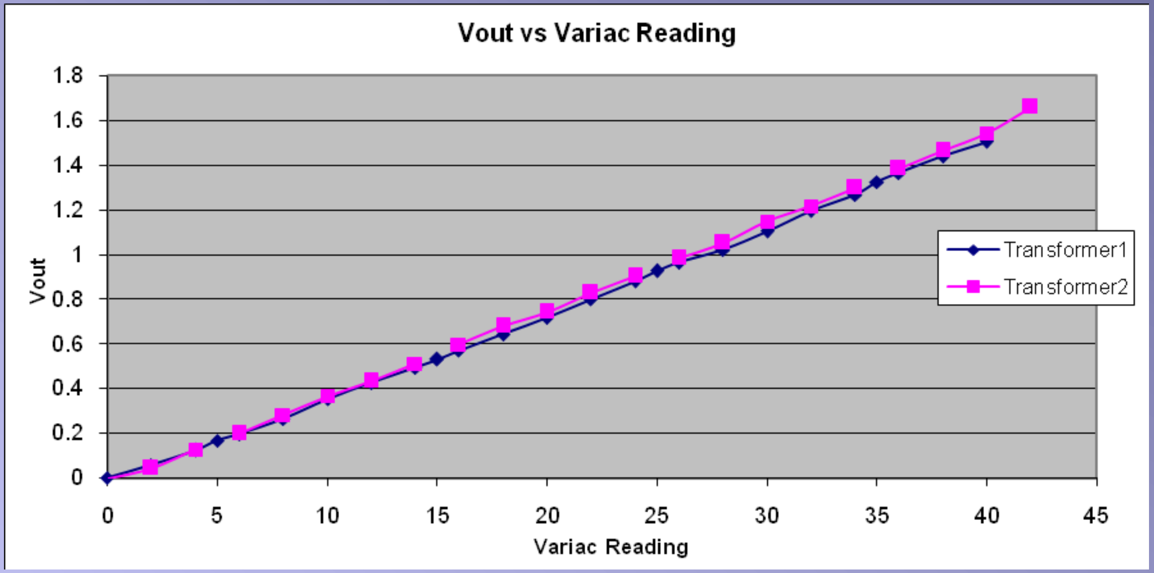
$$m = .386\,g$$

$$I = \frac{1}{12} m(d^2 + l^2)$$

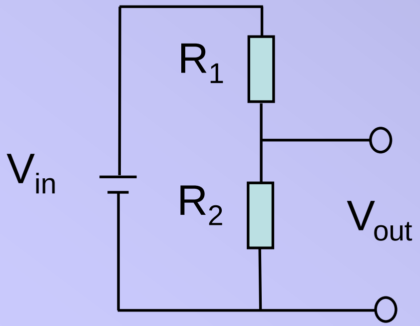
$$= 1.29x10^{-9}\ kg\ m^2$$

$$V = 3.28x10^{-8}\ m^3$$

Electric Field Measurement



Voltage divider:



$$R_1 = 98.55\ M\Omega$$
$$R_2 = 26{,}800\ \Omega$$

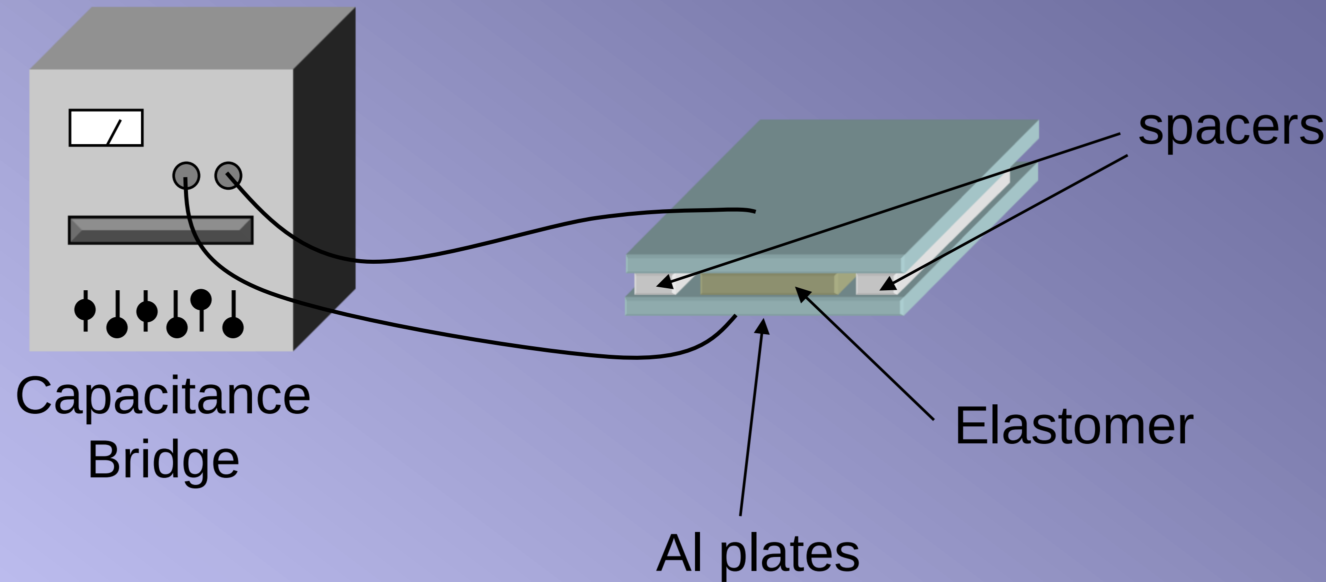
$$\longrightarrow$$

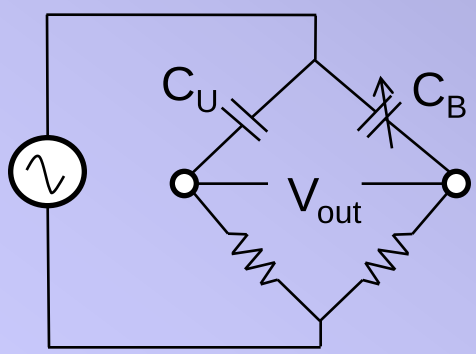
$$V_{in}(110) = 15{,}206\ V$$
$$V_{in}(140) = 19{,}353\ V$$

Thus

$$E_{app} = 5.95x10^5\ {}^{V}\!/_{m}$$

Experiment 2





Elastomer is sandwiched between aluminum plates and capacitance is measured with bridge

Theory & Results

Without elastomer:

$$C_{tot} = \frac{\varepsilon_s \varepsilon_0 A_s}{d} + \frac{\varepsilon_{air} \varepsilon_0 A_{air,1}}{d}$$

Using

$$\varepsilon_{air} = 1.00059$$

Find

$$\varepsilon_s = 3.88$$

With elastomer:

$$C_{tot} = \frac{\varepsilon_s \varepsilon_0 A_s}{d} + \frac{\varepsilon_{air} \varepsilon_0 A_{air,2}}{d} + \frac{\varepsilon_{\perp} \varepsilon_0 A_e}{d}$$

Find

$$\varepsilon_{\perp} = 3.38$$

Results

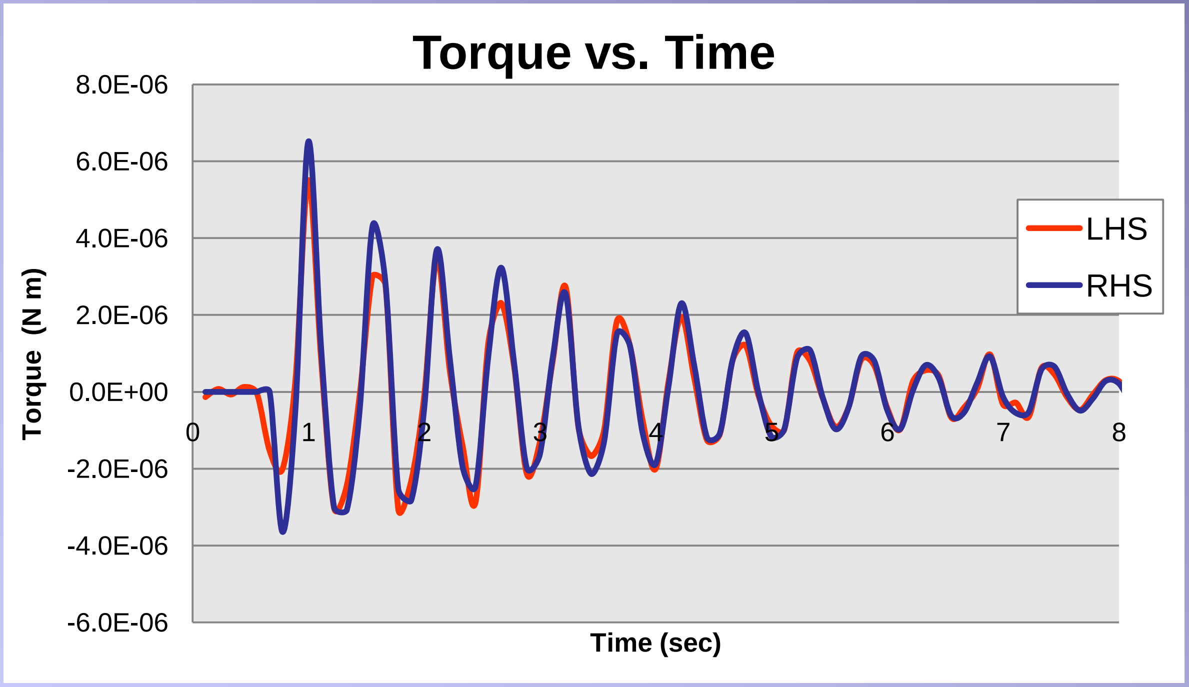
$$\tau = I\frac{d^2\theta}{dt^2} = \frac{d\xi}{d\theta}$$

LHS RHS

Using: $\varepsilon_{\parallel} = 3.29$

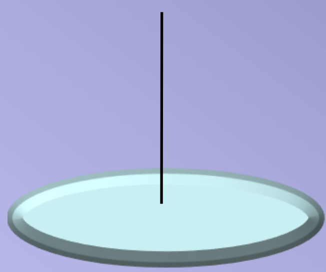
$\varepsilon_{\perp} = 3.38$

$k = 1.13x10^{-8}\ N/m$

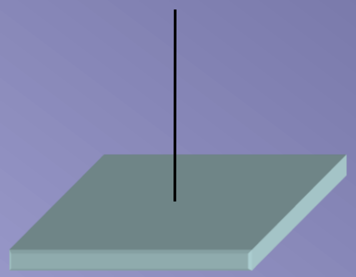


Possible Future Work

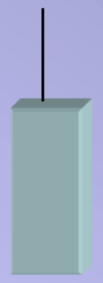
- Remove shape effects:



<u>OR</u>



- Remove anisotropy effects:



- Repeat experiments in magnetic field

Conclusions

- Electric field response used to determine elastomer dielectric constant values
- Would like to do similar magnetic field measurements
- Still a work in progress